Radiative Decays of Bosons

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A simplified model of the radiative decays of pseudoscalar and vector-mesons is investigated in the framework of a three-particle formalism for unstable composite particles in order to illustrate the role that unstable particles may play in providing information on the question of constituents of bosons.

1. Introduction

In previous works[1,2] attempts have been made to describe unstable fundamental particles starting from first principles. In this paper the application of the results[2] regarding the three-particle transition amplitude to radiative decays of current interest is presented.

The procedure adopted is the following. The relevant three-particle transition amplitudes which is factorized as the convolution of two-particle T-matrices, serve as a natural starting point, for expressing each T-matrix in terms of its bound state poles, a transition amplitude for composite particles is obtained. The amplitude is a function of the particle masses and coupling constants of the bound states which are, at least in principle, determined by the bound state problem.

2. The Framework

As framework for the investigation, consider the three-particle transition amplitude

$$\Delta_{kq\alpha,\, lp\beta} = - T_{ks,\, lr}\, G_{rj}\, T_{jq,\, fp}\, G_{f\lambda}\, T_{\lambda\alpha,\, \gamma\beta}\, G_{\gamma s} \tag{2.1}$$

which derives from a relativistic three-particle equation[2]. The indices in the last equation denote space-time coordinates, spin and isospin. Expressing each of the two-particle T-matrices at its bound state poles, the momentum-space transition amplitude involving a pseudoscalar meson and two vector-mesons reads

$$\begin{aligned} \Delta^{\mathrm{D}}_{kq\alpha,\, lp\beta} = & \{\gamma_5\, \varphi_u(P)\}_{kl} \left(\frac{g\, \hat{u}}{P^2 - m_u^2} \right) \\ & \cdot \{\gamma\, V_v(Q)\}_{pq} \left(\frac{g\, \hat{v}}{Q^2 - m_v^2} \right) \\ & \cdot \{\gamma\, V_w(R)\} \left(\frac{g_w}{R^2 - m_w^2} \right)_{\alpha\beta} \Delta^{\mathrm{t}} \end{aligned} \tag{2.2}$$

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where

$$\begin{aligned} \Delta^{\mathrm{t}}(P, Q, R) = & - (2\pi)^{-4}\, g_u\, g_v\, g_w \\ & \times \int \mathrm{d}^4 k\, \mathrm{Tr}[\gamma_5\, \varphi_u(P)\, G(k)\, \gamma\, V_v(Q) \\ & \times G(k-Q)\, \gamma\, V_w(R)\, G(k-P)] \,. \end{aligned} \tag{2.3}$$

In the last equation, Δ^{t} denotes the transition matrix element in terms of the incoming and outgoing composite particles of four-momenta P, Q and R respectively as indicated in Figure 1. The vertices are defined by the coupling constants of the particles to nucleons as follows:

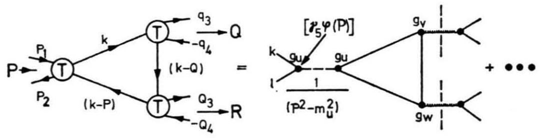

Fig. 1. The three-particle transition matrix element for composite particles depicted as a factorized product in terms of the two-particle T-matrix and its decomposition in the proximity of two-particle bound state poles.

(a) Pseudoscalar coupling

$$H_{\mathrm{int.}} = G\, (\bar{p}\, \gamma_5\, p)\, \pi^0 \,. \tag{2.4}$$

(b) Electromagnetic coupling

$$H_{\mathrm{int.}} = e\, (\bar{p}\, \gamma^\mu\, p)\, A_\mu \,. \tag{2.5}$$

(c) Vector-meson coupling

$$\begin{aligned} H_{\mathrm{int.}} = & Q_1\, (\bar{p}\, \gamma^\mu\, p)\, \varrho_\mu + Q_2\, (\bar{p}\, \gamma^\mu\, p)\, \Phi_\mu \\ & + Q_3\, (\bar{p}\, \gamma^\mu\, p)\, \omega_\mu + \cdots \end{aligned} \tag{2.6}$$

which in the SU(3) limit may be expressed by

$$\begin{aligned} H_{\mathrm{int.}} = & \frac{f}{2}\, [(\bar{p}\, \gamma^\mu\, p)\, \varrho_\mu + \sqrt{3}\, (\bar{p}\, \gamma^\mu\, p)\, \Phi_\mu] \\ & + \tfrac{1}{2} \sqrt{3} \gamma\, (\bar{p}\, \gamma^\mu\, p)\, \omega_\mu + \cdots \,. \end{aligned} \tag{2.7}$$

3. Radiative Decays of Pseudoscalar Mesons

In this section the radiative decays of pseudoscalar mesons ($P = \pi^0, \eta, \eta', E$-meson)

$$P \to \gamma_1 + \gamma_2 \tag{3.1}$$

are considered. Let p_1, p_2 and ε_1, ε_2 denote the four-momenta and the polarization vectors of the photons respectively.

The most general kinematically allowed form of the amplitude is given by

$$F\, \varepsilon_{\mu\nu\varrho\sigma} Q^\mu R^\nu \varepsilon_1^\varrho \varepsilon_2^\sigma \,. \tag{3.2}$$

The presence of the completely antisymmetric tensor is a consequence of the pseudoscalar nature of the parent meson. It has been known for some time, that in perturbation theory the „effective-coupling" of the neutral pion to the electromagnetic field is proportional to[3]

$$F = -\, e^2 G/4\pi^2 M \tag{3.3}$$

treated in the soft-pion limit i.e. to lowest order in (m/M). In the above m denotes the pion mass and M the proton mass. The local coupling expressed by Eq. (3.3) is necessarily an approximation to the effective non-local interaction.

It is shown that our three-particle approach based on Eqs. (2.1) and (2.3) substantiates this result and that the dynamical content of the decay based on the vertex mechanism can be carried out without invoking any approximations. In particular, it is found that

$$F = -\frac{e^2 G M}{\pi^2 m^2}\left[\arcsin\left(\frac{m}{2M}\right)\right]^2 . \tag{3.4}$$

The non-locality is more pronounced for heavier pseudoscalar mesons. It would also be of interest to investigate its effect on off-shell amplitudes such as in the $K_L \to 2\gamma$ decay, Primakoff-effect and the neutral pion pole contribution to photon-photon scattering. Before carrying out the analysis, a remark on the composite nature of the particles is in order. This aspect is brought out quite clearly by the study[1] of the radiative decays in the unified theory of elementary particles. The analytical properties of the vertex mechanism hinge on the spectral representations of the two-point function and Green's function and is tied up with the bound state problem itself. In this approach the particles are pictured as fermion-antifermion bound states in which leptons and gauge quanta play a substantial role. The present investigation is not spread on such a broad canvas. The bound state problem is implicit in the factorization of the T-matrix in terms of the bound state poles regarded as nucleon-antinucleon bound states.

The matrix element Δ^t for the radiative decay of a pseudoscalar meson assumes the form

$$\Delta^t = -\frac{2i\, e^2 G}{(2\pi)^4} \times \int d^4k \frac{\mathrm{Tr}[\gg]}{[k^2 - M^2][(k-P)^2 - M^2][(k-Q)^2 - M^2]} \tag{3.5}$$

The trace which occurs in the amplitude is given by

$$\begin{aligned}\mathrm{Tr}[\gg] &= \mathrm{Tr}[\gamma_5 \{\gamma k + M\}\gamma \varepsilon_1 \{\gamma(k-Q) + M\}\gamma \varepsilon_2 \\ &\quad \times \{\gamma(k-P) + M\}] \\ &= 4M\, \varepsilon_{\mu\nu\varrho\sigma} Q^\mu R^\nu \varepsilon_1^\varrho \varepsilon_2^\sigma \,.\end{aligned} \tag{3.6}$$

The amplitude as it stands is logarithmically divergent. However, as a consequence of the symmetry properties of the particles participating in the decay, the amplitude is well defined as shown by Equation (3.6). Employing standard techniques, the contribution of the vertex diagram may be expressed in the form

$$R = -\frac{e^2 G \Lambda}{2\pi^2 M}\int_0^1 dx \int_0^1 \frac{dy\, y}{[1 - \varrho(1-x)\, y(1-y)]} \tag{3.7}$$

where

$$\varrho = m^2 M^{-2} \tag{3.8}$$

and

$$\Lambda = \varepsilon_{\mu\nu\varrho\sigma} Q^\mu R^\nu \varepsilon_1^\varrho \varepsilon_2^\sigma \,. \tag{3.9}$$

The last equations yield the result

$$R = -\frac{e^2 G \Lambda}{\pi^2 M}\int_0^1 \frac{dx}{[4A - A^2]^{1/2}} \arctan\left[\frac{A^{1/2}}{[4-A]^{1/2}}\right] \tag{3.10}$$

where

$$A = \varrho(1-x)\,.$$

After straightforward manipulations the matrix element may be cast in the form

$$R = -\frac{2e^2 G \Lambda M}{\pi^2 m^2}\int_0^a \frac{d\lambda}{[1-\lambda^2]^{1/2}} \arcsin(\lambda) \tag{3.11}$$

where

$$a = (m/2M)\,. \tag{3.12}$$

Therefore

$$\Delta^t = -\frac{e^2 G \Lambda M}{\pi^2 m^2}\left[\arcsin\left(\frac{m}{2M}\right)\right]^2 . \tag{3.13}$$

After performing the summation over the polarization of the photons, evaluating the phase-space factor and including the double polarization degeneracy of the photons, the decay width equals

$$\Gamma(P \to 2\gamma) = \left(\frac{e^2}{4\pi}\right)^2 \left(\frac{G^2}{4\pi}\right) \frac{M^2}{\pi^2 m}\left[\arcsin\left(\frac{m}{2M}\right)\right]^4 . \tag{3.14}$$

4. Radiative Decays of Vector-Mesons

The matrix element of the radiative decay

$$V \to P + \gamma \tag{4.1}$$

of a vector meson (regarded as a fermion-antifermion bound state[5]) is given by

$$\Delta^{t} = -2 i Q e G (2\pi)^{-4} \times \int d^4 k \frac{\mathrm{Tr}[\gg]}{[k^2 - M^2][(k-Q)^2 - M^2][(k-P)^2 - M^2]} \tag{4.2}$$

where

$$\mathrm{Tr}[\gg] = \mathrm{Tr}[\gamma V \{\gamma k + M\} \gamma \varepsilon \times \{\gamma (k-Q) + M\} \gamma_5 \{\gamma (k-P) + M\}] \tag{4.3}$$

and V and ε denote the polarization vectors of the vector-meson and photon respectively. Employing the procedure of the previous section it follows that

$$\Delta_t = -Q e G \Lambda_V M \pi^{-2} (M_V^2 - m^2)^{-1} J \tag{4.4}$$

where

$$J = \left[\arcsin\left(\frac{M_{\hat{V}}}{2M}\right)\right]^2 - \left[\arcsin\left(\frac{m}{2M}\right)\right]^2 \tag{4.5}$$

$$\Lambda_V = \varepsilon_{\mu\nu\varrho\sigma} Q^\mu P^\nu \varepsilon^\varrho V^\sigma \tag{4.6}$$

and M_V denotes the vector-meson mass. Summing over the final polarization of the photon and averaging over the vector-meson polarization yield

$$\tfrac{1}{3} \Sigma |\Lambda_V|^2 = \tfrac{1}{6} (M_V^2 - m^2)^2 . \tag{4.7}$$

The decay width for the vector-meson radiative decay is therefore given by

$$\Gamma(V \to P + \gamma) = \frac{2}{3}\left(\frac{e^2}{4\pi}\right)\left(\frac{Q^2}{4\pi}\right)\left(\frac{G^2}{4\pi}\right)\frac{M^2}{\pi^2 M_{\hat{V}}} \times \left(1 - \frac{m^2}{M_V^2}\right)\left|\left\{\arcsin\left(\frac{M_{\hat{V}}}{2M}\right)\right\}^2 - \left\{\arcsin\left(\frac{m}{2M}\right)\right\}^2\right|^2 . \tag{4.8}$$

5. Applications and Conclusions

The partial decay widths for the radiative decays of neutral pseudoscalar- and vector-mesons calculated from Eq. (3.14) and (4.8) are given in Table 1 and denoted by Γ.

The results are compared to the experimental and quark model results. The symbol α, in the table denotes

$$\alpha = (4\pi)^{-1} Q_\omega^2 = (137.06)^{-1} .$$

It is found that the model provides a satisfactory description of the radiative decays under certain provisos for the coupling constants.

The decays which involve unknown coupling constant are given in the following equations

$$\Gamma(\eta' \to 2\gamma) = \left(\frac{G^2{}_{\eta' NN}}{4\pi}\right) 0.41\ \mathrm{keV}, \tag{5.1}$$

$$\Gamma(E(1424) \to 2\gamma) = \left(\frac{G^2{}_{ENN}}{4\pi}\right) 1.84\ \mathrm{keV}, \tag{5.2}$$

$$\Gamma(\Phi_\mu \to \eta' + \gamma) = \left(\frac{G^2{}_{\eta' NN}}{4\pi}\right) 0.14\ \mathrm{keV}, \tag{5.3}$$

for

$$G_{\Phi NN}^2 / 4\pi = 1.575$$

and

$$\Gamma(\Phi_\mu \to \eta' + \gamma) = \left(\frac{G^2{}_{\eta' NN}}{4\pi}\right) 0.24\ \mathrm{keV} \tag{5.4}$$

for

$$G_{\Phi NN}^2 / 4\pi = 2.75 .$$

The results of the table indicate that from the decay widths measured to date, no clearcut preference is in evidence for picturing bosons as predominantly nucleon-antinucleon systems or the

Table 1

Decay	$g_u^2/4\pi$	$Q_v^2/4\pi$	Γ	experimental[e]	quark model[f]
$\pi^0 \to \gamma + \gamma$	14.1[a]	α	13.3 eV	(7.3 ± 1.2) eV	12 eV[h]
$\pi^0 \to \gamma + \gamma$	12.5[b]	α	11.8 eV	(11.7 ± 1.2) eV[g]	12 eV
$\eta \to \gamma + \gamma$	8[c]	α	0.53 keV	(1.0 ± 0.23) keV	0.45 keV[h]
$\eta \to \gamma + \gamma$	12.5	α	0.83 keV	(1.0 ± 0.23) keV	0.45 keV
$\omega \to \pi + \gamma$	3.7[d]	14.1	0.91 MeV	(1.1) keV	1.17 MeV
$\varrho \to \pi + \gamma$	0.525[a]	14.1	0.12 MeV		0.12 MeV
$\varphi \to \pi + \gamma$	1.575[c]	14.1	0.98 MeV		0
$\varphi \to \pi + \gamma$	2.75[d]	14.1	1.71 MeV		0
$\omega \to \eta + \gamma$	3.7	12.5	120 keV		6.4 keV
$\omega \to \eta + \gamma$	3.7	8	78 keV		6.4 keV
$\varphi \to \eta + \gamma$	1.575	8	0.22 MeV		0.34 MeV
$\varphi \to \eta + \gamma$	2.75	12.5	0.61 MeV		0.34 MeV
$\varphi \to \eta + \gamma$	2.75	8	0.39 MeV		0.34 MeV
$\varphi \to \eta + \gamma$	1.575	12.5	0.34 MeV		0.34 MeV
$\varrho \to \eta + \gamma$	0.525	12.5	14 keV		50 keV
$\varrho \to \eta + \gamma$	0.525	8	9 keV		50 keV

[a] See Ref. 4. [b] See e. g. Ref. 5. [c] SU(3) result. [d] See Ref. 6. [e] For experimental data see Ref. 7. [f] See Ref. 8. [g] See Ref. 9. [h] See also Ref. 10.

suggestion that fundamental particles may be pictured as fabricated from quarks. It is apparent that a measurement of the decays $\varrho \to \eta + \gamma$ and $\omega \to \eta + \gamma$ are most important in this respect.

In summary, a simplified model has been investigated to illustrate the role which unstable particles may play in investigating the constituents of fundamental particles.

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Repulsive Interaction Potential between Rare-Gas Atoms in the Fermi-Amaldi Model

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Using an approximate expression based on the Fermi-Amaldi statistical model of the atom, the repulsive interaction energies $U(R)$ between a pair of nenutral rare-gas atoms have been derived. This paper contains a comparison of our numerical results for some repulsive interaction potentials between rare-gas atoms with the corresponding TFD results obtained by Abrahamson.

This paper concerns a derivation of the theoretical expression for a repulsive interaction potential between rare-gas atoms using the Fermi-Amaldi [1] model. The interaction potential was calculated with the so called screened Coulomb potential due to Bohr [2], i. e.

$$U(R) = (Z_1 Z_2 e^2/R) \exp\{-R/a'\} \tag{1}$$

where e is the magnitude of the electronic charge; Z_1, Z_2 are respective atomic numbers of the interacting atoms; and the screening length a' is defined as

$$a' = a_0/(Z_1^{2/3} + Z_2^{2/3})^{1/2} \tag{2}$$

with a_0 ($=0.529$ Å) denoting the first Bohr radius in hydrogen. It has been found [3] that this interaction is fairly realistic at very small internuclear distances R, but less satisfactory elsewhere [4]. Another theoretical expression based on the Thomas-Fermi (TF) statistical model of the atom [5] is given by Firsov [6].

This model is applicable for small R but it gives unrealistically big values for $U(R)$ if R increases. A third theoretical potential for the repulsive interaction for the rare-gas atoms is based on the Thomas-Fermi-Dirac model (TFD). This model was considered by Abrahamson [7] in a very detailed way. The numerical calculations show that the agreement with experiment is closer and more extensive than it was previously estimated.

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